IS SELF-ORGANIZED CRITICALITY POSSIBLE IN DRY FRICTION?

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Abstract. Various experiments and model simulations are reviewed which were carried out in order to find whether irregular stick-slip motion in dry friction fits into the framework of self-organized criticality.

1. Introduction

It is well-known that stick-slip motion can be very irregular [1]. Is it possible to understand these irregularities without going too much into the details of the complex interaction between two sliding surfaces?

In 1987 Bak, Tang, and Wiesenfeld proposed a new type of irregular behavior which they called *self-organized criticality* (SOC) [2]. It should appear in many open systems which are driven far away from thermal equilibrium. This new idea has triggered a lot of research interests. For example, a geophysical model of an earthquake fault (the Burridge-Knopoff model) was reinvestigated from this novel point of view [3]. Later on the first friction experiment was carried out [4] in order to find out whether erratic stick-slip motion is a manifestation of SOC.

In the next section I briefly explain SOC. Sec. 3 and 4 review the main results from four experiments and from numerical studies of four different models, respectively. This contribution ends with concluding remarks.

2. Self-Organized Criticality

The idea behind SOC is that a dissipative (i.e., open) system with many effective degrees of freedom may organize itself into a critical state if it is driven very slowly away from thermal equilibrium. Here "critical" is meant in the sense of a second-order phase transition where strong fluctuations and non-trivial power laws are characteristic. It can also interpreted in the sense of non-linear dynamics where a critical state is neither stable nor unstable.

SOC is often illustrated by the *sand-pile metaphor*. Imagine a sand-pile on which grains of sand are put at a very slow rate. If the pile is flat nothing interesting will happen. Only the slope of the pile slowly increases. If the slope reaches a critical value avalanches of any sizes will take place. The slope remains at the critical value instead of increasing further. Due to the avalanches the continuous rate of grains is transformed into a strongly fluctuating flow at the bottom of the pile. The distribution of the size of avalanches as well as the power spectrum of the flow show power laws. The sand-pile metaphor is only an illustrative picture. Real sand-piles usually do not behave in this way.

The concept of SOC has become very attractive but it was not introduced as a precisely defined concept. Therefore many experimental studies and theoretical models are sold as examples of SOC which are not belonging to this class of systems [5]. In the following the key features of SOC are listed:

1. The system should have a *huge* number of meta-stable states, that is, their number should increase exponentially with the number of subsystems (e.g. grains of sand).
2. The system should be driven *very slowly* away from the ground state.
3. There should be recognizable relaxation events (e.g. avalanches). The energy which is put into the system due to the driving is released in these events. The driving has to be very slow in order to separate the time scales averaged duration time of an event and averaged waiting time for the next event.
4. An important feature is that the distribution of the event strength should be a power law. This means that *all* events stronger than some value should be power-law distributed except those which are feeling the size of system. Furthermore the system-spanning events should not dominate the averaged value of the event strength.
5. The power spectrum of a characteristic property (e.g. the flow at the bottom of the sand-pile) should be of the form $1/f^B$ with an exponent B which is not trivial, i.e., not an even integer.
6. After a possible transient all statistical properties should be *stationary* in the asymptotic time limit. This means that the system has reached a non-trivial *attractor* which is a typical sign of self-organization.
7. The behavior should be to a large extent independent of the initial conditions.
8. There should be no fine tuning of system parameters except for the

driving strength which should be infinitesimal small in the ideal case[1].

Can we expect SOC in dry friction? The sliding of two surfaces in contact is an open system which is driven far away from equilibrium. The very slow driving is realized in the limit of sliding velocity $v \to 0$. The slips in a stick-slip motion are the recognizable events which can be quantified by their slip size. Thus stick-slip motion in dry friction is a potential candidate for SOC.

But the crucial question is: How many meta-stable states do exist? There should be a huge number of them for a given distance between the centers of mass of the sliding bodies. This is possible only if the stiffness of at least one body is weak enough to allow deformations in such a way that a single asperity can lock at different places. Furthermore, the number of locked asperities should be relatively large. It is usually easy to control the number of meta-stable states in models but it is often hard to figure it out in a real experiment.

3. Experiments

The first dry friction experiment inspired by the new concept of SOC was done by Feder and Feder [4] where a piece of sand-paper attached to a rigid block is moved over a carpet. For the slip-size distribution they found a power law over two decades with an exponent of roughly 1.8. There was an exponentially like cutoff but the authors did not checked whether this cutoff depends on the system size or not. They also did not report about the stationarity of the statistics and the dependence of the results on system parameters like the load, the stiffness of the machine etc. The power spectrum of the stick-slip curve was a trivial $1/f^2$ law which reflects the power spectrum of the step function caused by each slip. This experiment shows that SOC might be possible in dry friction. But it does not proof the possibility because a lot of key features has not been checked by the authors.

In the experiment of Vallette and Gollub [6] a stretched latex membrane is moved over a long cylindrical glass rod. This is a quasi-one-dimensional system which was designed to test the Burridge-Knopoff model (see next section). The distribution of the slip size decreases for increasing slip size but it was not possible to extract a power law. For larger slip sizes the distribution increases and forms a bump. This feature was also found in simulations of the Burridge-Knopoff model [3]. The system clearly does not organize itself into a critical state.

[1]The ideal case assumes zero temperature and zero quantum fluctuations. Otherwise the system would be always in the ground state. In the precence of fluctuations the driving has to be finite but as small as possible.

Johansen *et al.* [7] investigated a traditional tribological experiment where a steel block slides on a rotating steel table. The block is connected to a fixed support via a spring. The slip-size distribution depends on the load and the stiffness of the spring. Apart from exponential and gaussian distributions they found a power-law distribution only in a case of a small load and a stiff spring. The power law extends only over one decade (exponent 1.6) followed by a typical exponential cutoff. It is not clear whether this a finite size cutoff or not. The statistics seemed to be stationary and independent of the initial conditions. The question about the fine tuning is difficult to answer. On one hand the authors found the power-law only for one configuration of loads and spring stiffnesses. On the other hand they have investigated only three different configurations. The power spectrum of the stick-slip curve did not show a power-law.

The strongest evidence for SOC was found in an experiment done by Ciliberto and Laroche [8] where the sliding surfaces have a well-defined artificial roughness and elasticity. The roughness is created by macroscopic steel spheres (diameter 2 mm). On one surface these spheres are rigidly mounted whereas on the other surface they are embedded into silicon rubber. The spheres in the rubber are able to move in the sliding plane but not in the direction perpendicular to that plane. The authors found clear power-laws (exponents between 1.5 and 2) over 1.5 decades with an exponential cutoff. By increasing the load or the size of the system the cutoff shifts to higher values. Thus it is really a finite-size cutoff. They also found a power spectrum with a nontrivial power-law (exponent 3). In the one-dimensional version of this experiment the authors found the same kind of bump for large slips as observed in the Vallette-Gollub experiment.

4. Models

In this section the results of four different models of dry-friction systems with many degrees of freedom are briefly reviewed.

4.1. THE BURRIDGE-KNOPOFF (BK) MODEL

In 1967 Burridge and Knopoff introduced a model of an earthquake fault [9]: A chain of N blocks with a harmonic next-neighbor interaction represents a lubricant between two sliding surfaces (see Fig. 1). The interaction with the lower surface is modeled by a phenomenological dry-friction law whereas the upper surface is connected with each block via springs. The upper surface is pulled with a very slow but constant sliding velocity v.

In 1989 and 1991 Carlson *et al.* [3, 10] reinvestigated this model. In their version of the BK model all blocks have the same mass, the next neighbor springs are identical and the all springs connecting the blocks with the

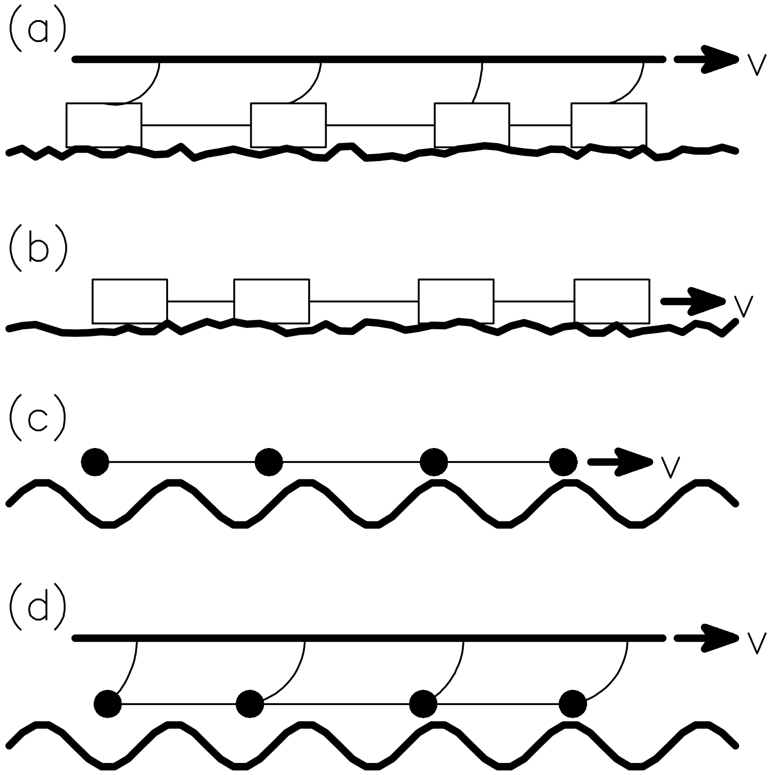

Figure 1. The Burridge-Knopoff (BK) model (a), the train model (b), the Frenkel-Kontorova (FK) model (c), and the Frenkel-Kontorova-Tomlinson (FKT) model (d).

upper surface have the same stiffness. Also the friction law is the same for all blocks. Thus the equation of motion reads in dimensionless units:

$$\ddot{x}_j + F_{fric}(\dot{x}_j) = x_{j+1} - 2x_j + x_{j-1} + \kappa(vt - x_j), \quad j = 1, \cdots, N. \qquad (1)$$

where κ is the ratio of the stiffnesses of the two different types of springs in the model. Carlson *et al.* have chosen the velocity-weakening friction law

$$F_{fric}(v) = \frac{\text{sign}\,(v)}{1 + 2\alpha|v|}, \qquad (2)$$

which is a special case of the friction force introduced in [9]. If a block slides it will feel this kinetic friction force. Carlson *et al.* have assumed that the static friction is identical with (or slightly above [10]) $F_{fric}(v \to 0) = 1$ which means that a block at rest will be at rest as long as the absolute value of the right hand side of (1) is less than one.

The number of meta-stable states is huge. In fact, it is infinite because the finite static friction allows each block to vary its position in a finite interval.

If the resting chain is pulled very slowly the upper springs will be slowly stretched which leads to an increase of the force acting at each block. For a certain block the static friction will be eventually exceeded and the block starts to slip. It accelerates because the kinetic friction is less than the static one. Later on the velocity decreases and the block stops after the velocity has reached zero because the sum of the forces caused by the springs will be usually less than the static friction. Due to the next-neighbor interaction the neighbors of the sliding block may also start to slide. This leads to an earthquake (i.e., slipping event) which may involve the whole chain. Its strength can be measured by the sum of the displacements

$$S = \sum_{j=1}^{N} |x_j^{after} - x_j^{before}|. \tag{3}$$

Since $x_j^{after} > x_j^{before}$ is nearly always true, S/N is the displacement of the center of mass, i.e., the total slip size.

For weak upper springs ($\kappa = 0.01$) slips of medium size ("localized events" in the terminology of Carlson *et al.*) are power-law distributed (exponent 1). Large slips (called "delocalized events") are much more probable than what is expected from the power law. This leads to a bump in the distribution. The cross-over point $\tilde{S}$ between localized and delocalized events is independent of N and scales like $\kappa^{-1/2}$. There is also an N-independent cutoff S^* for the delocalized events which scales like $1/\kappa$.

The distribution of the slip length L (i.e., the number of blocks which slip during an event) is similar. The corresponding cross-over point is $\tilde{L}$ which also scales like $\kappa^{-1/2}$. The length of the largest slip is independent of N (if N is large enough) which means that in very large systems the largest slip affects only a small portion of the chain.

Thus the one-dimensional BK model does *not* organize itself into a critical state. This is confirmed by the above mentioned experiments [6, 8] which are inspired by the studies of Carlson *et al.* The Ciliberto-Laroche experiment [8] suggests that the dimensionality is important. Carlson has studied a two-dimensional BK model but with block displacements only in one direction [11]. Qualitatively, she found the same results as for the one-dimensional case. Only the bump in the distribution is weaker but it is still there. It is not a finite-size effect because the largest events do not span the entire system.

SOC is suppressed in the BK model because of an intrinsic length scale. This length scale can be obtained from the continuous limit of the BK model which leads to the Klein-Gordon equation $\ddot{u} = u'' - \kappa u$ plus a non-linear friction term. The block index j is interpreted as a continuous space variable. The stiffness ratio κ can be absorbed by the space coordinate which leads to the rescaling $j \to j/\sqrt{\kappa}$. Therefore one understands why the cross-over point $\tilde{L}$ scales like $\kappa^{-1/2}$.

4.2. THE TRAIN MODEL

The train model is a variant of the BK model where the coupling to the upper surface is removed. Instead of driving the chain uniformly only the first block is pulled with a constant velocity v (see Fig. 1). This model was already introduced by Burridge and Knopoff [9]. They did an experimental studies with eight blocks where they found a power-law (exponent roughly 2) for the distribution of the slip size measured in terms of the change of potential energy. Here I briefly describe the result of a numerical study with much more blocks ($N = 100$) done by de Sousa Vieira [12].

The equation of motion is (1) with $\kappa = 0$ and $x_0 = vt$. De Sousa Vieira has chosen the same friction law (2) as Carlson *et al.*. In the train model slipping events always start with the slip of the first block directly behind the driving block. The distribution of the slip size S defined by (3) shows a clear power law over at least three decades with an exponent between 1.4 and 1.6 depending on α. The power law is limited only by a finite size cutoff. Also the slip length L (i.e., the number of blocks which are involved in an event) is power-law distributed (exponent between 1.6 and 2.2). This behavior is robust and does not depend on the initial conditions. It only fails if α is below a critical value.

The train model organizes itself into a critical state. This is no surprise because the train model is in some sense the $\kappa \to 0$ limit of the BK model. Thus the characteristic length scale $\tilde{L}$, which can be interpreted as a correlation length, diverges and the power-law regime extends to infinity.

How important is the type of friction F_{fric} for the overall qualitative behavior of the BK model and the train model? All studies mentioned above are done with the friction law (2) introduced by Carlson *et al.* I expect that the behavior is robust and independent of the details of the friction law. Especially it has not to be velocity-weakening. Presumably any friction law works where for small velocities the kinetic friction is considerably below the static friction.

4.3. THE FRENKEL-KONTOROVA (FK) MODEL

In the two last sub-sections I will briefly discuss the results of my own studies of models which are similar to the BK model and the train model. The difference is that the phenomenological friction law F_{fric} is replaced by a kind of "microscopic" realization of such a friction law (see Fig. 1):

$$F_{fric}(\dot{x}_j) \to g\dot{x}_j + \frac{dV_E}{dx}(x_j), \tag{4}$$

where g is a damping constant and V_E is a periodic potential. The blocks are interpreted as point particles which are pinned by the potential V_E. The maximum of dV_E/dx can be interpreted as a static friction force. Due to inertia (i.e., if g is small enough) a particle can slide over the maxima of the external potential even for forces below the static friction force. Thus the effective kinetic friction will be lower than the static friction. One therefore expects similar behavior as in the BK model and variants.

In my first study [13] I have investigated the counterpart of the train model. The equation of motion in dimensionless units reads

$$\ddot{x}_j + g\dot{x}_j = x_{j+1} - 2x_j + x_{j-1} - V_E'(x_j), \quad j = 1, \cdots N, \tag{5}$$

with $x_0 = vt$ and $x_{N+1} = x_N - a$. Here a is the equilibrium length of the springs. The undriven system is the Frenkel-Kontorova (FK) model which is well-known in the field of commensurate-incommensurate transition [14] and of charge-density waves (see e.g. [15]).

In order to realize infinitesimally small driving (i.e., $v \to 0$) I have chosen a piecewise parabolic potential

$$V_E(x) = b\left(\frac{1}{2} - x \bmod 1\right)^2. \tag{6}$$

For such a potential a stationary solution can be calculated from knowing only the potential well in which each particle sits. Furthermore, the stability interval for x_0 can be calculated and the loading can be computationally abridged (for details see Ref. [13]) which makes infinitesimal slow driving possible. The number of meta-stable states is really huge because it scales like η^N, where $\eta = 1 + b + \sqrt{b(b+2)}$.

As in the train model slipping events always start at the pulled end of the chain. After some transient (it usually ends after the first slip involving the whole chain) the system reaches a kind of chaotic attractor which is characterized by the fact that the difference of the lengths of two neighboring springs is on average constant along the chain, i.e.,

$$\langle f \rangle \equiv \langle x_{j+1} - 2x_j + x_{j-1} \rangle = \text{const}, \tag{7}$$

where $\langle\rangle$ denotes the time average. I call f the *local stress*. From (7) follows $\langle E_{pot} \rangle = N^3[\langle f \rangle^2/6 + \mathcal{O}(1/N)]$.

Figure 2 shows the cumulative densities $p(S)$ and $p(L)$ for the slip size S and slip length L, respectively. The slip size is defined by (3). The slip

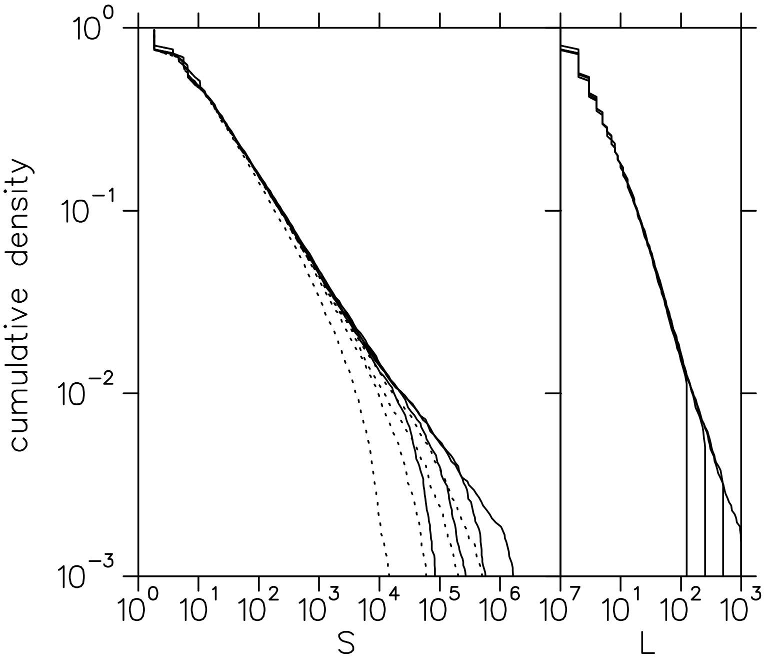

Figure 2. The FK model: Cumulative densities of the slip size S and the slip length L for four different system lengths $N = 125$, 250, 500, and 1000. Parameters: $a = 1.3$, $b = 5$, $g = 0.1$. Each curve is the result of a simulation with $2 \cdot 10^4$ slip events. The solid (dotted) lines in left part of the figure denote the statistics with (without) system-spanning slips (i.e., slips with $L = N$).

length is the number of particles which change their potential well during an event. The cumulative density $p(S)$ is the probability to find a slip with a size larger than S. The derivative of a cumulative density defines the distribution $\rho(S) = -dP/dS$ which is a probability density. A power law with exponent B for the cumulative density leads to a power law for the distribution with an exponent $B + 1$. On the other hand, a power law distribution with an exponent less than or equal to one do not lead to a power law for the cumulative density.

The cumulative densities $p(S)$ in Fig. 2 show clear power laws with cutoffs which shift to larger values if N is increased. A careful analysis of the data shows that one has to distinguish between slips which do not reach the end of the chain and slips with $L = N$, so-called system-spanning slips. The crucial question is now: Do the system-spanning events dominate the averaged slip size $\langle S \rangle$ in the thermodynamic limit $N \to \infty$? If the answer is positive the locally driven FK model will *not* organize itself into a critical state even if the distribution of the non-system-spanning slips is a power

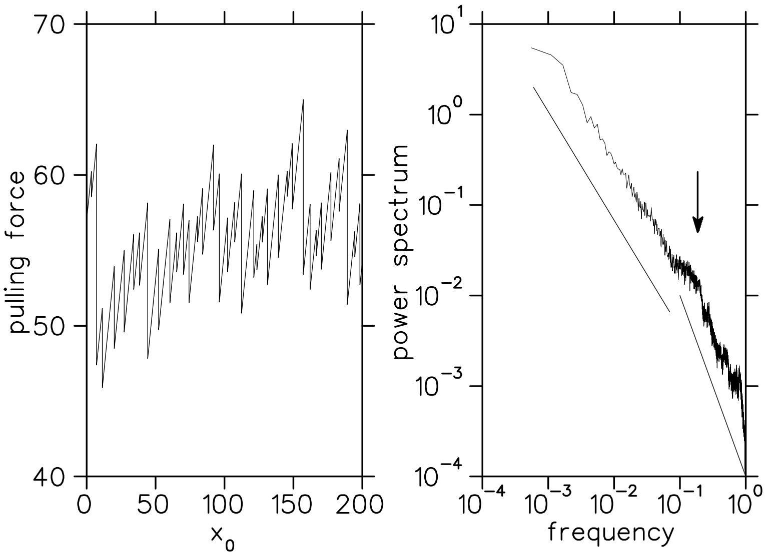

Figure 3. The FK model: The pulling force and its power spectrum for $N = 125$, $a = 1.3$, $b = 5$, $g = 0.1$. The slope in the low frequency regime is -1.2. The arrow denotes the frequency corresponding to the mean distance between successive jumps in the pulling force. The slope in the high frequency regime is -2. It is a signature of these jumps.

law.

From data summarized in Fig. 2 I found that the probability $p(L = N)$ of a system-spanning slips scales like $1/N^\nu$ with $\nu = 0.95 \pm 0.05$. The averaged slip size $\langle S\rangle_{L=N}$ of such slips scales like N^γ with $\gamma = 1.87 \pm 0.05$. The cumulative densities $p_{L<N}(S)$ of the non-system-spanning slips (the solid lines in Fig. 2) can be described by an N-independent scaling function G:

$$p_{L<N}(S) = S^{-B}G(S/N^\alpha), \quad \text{with} \quad B = 0.54 \pm 0.02 \text{ and } \alpha = 2.5 \pm 0.2. \tag{8}$$

From this scaling one gets for the averaged slip size

$$\langle S\rangle_{L<N} = \int_{S_{min}}^{\infty} S\left[-\frac{d}{dS}p_{L<N}(S)\right] dS = C_0 N^{\alpha(1-B)}, \tag{9}$$

where C_0 is N-independent for large N. With B and α obtained from (8) the scaling exponent of $\langle S\rangle_{L<N}$ becomes

$$\alpha(1 - B) = 1.15 \pm 0.14. \tag{10}$$

From direct measurements of $\langle S\rangle_{L<N}$ I got

$$\alpha(1 - B) = 1.19 \pm 0.03, \tag{11}$$

which is consistent with (10). The total averaged slip size

$$\langle S\rangle = [1-p(L=N)]\langle S\rangle_{L<N} + p(L=N)\langle S\rangle_{L=N} \tag{12}$$

has two contributions. In the first term $p(L=N)$ can be dropped for large N. It then scales like $N^{\alpha(1-B)}$. The second term scales like $N^{\gamma-\nu}$, with

$$\gamma-\nu = 0.92 \pm 0.10. \tag{13}$$

If $\gamma-\nu > \alpha(1-B)$ the system-spanning slips would dominate $\langle S\rangle$ because of

$$Q \equiv \frac{p(L=N)\langle S\rangle_{L=N}}{\langle S\rangle_{L<N}} = \frac{\langle S\rangle}{\langle S\rangle_{L<N}} - 1 \sim N^{\gamma-\nu-\alpha(1-B)}. \tag{14}$$

Here $\gamma-\nu$ is below $\alpha(1-B)$ but the error bars are just overlapping, i.e., $\alpha(1-B)-(\gamma-\nu) = 0.23 \pm 0.24$. By measuring $\langle S\rangle$ and $\langle S\rangle_{L<N}$ directly I found a clearly *decreasing* Q with a scaling exponent

$$\alpha(1-B)-\gamma+\nu = 0.27 \pm 0.03. \tag{15}$$

From this analysis one would conclude that system-spanning slips do not dominate the behavior in the thermodynamic limit. But the results are in contradiction to the sum rule

$$\langle S\rangle = \Delta x_0 N, \tag{16}$$

where Δx_0 is the mean difference of two successive positions of the zeroth particle where slips occur. Thus $\max(\alpha(1-B), \gamma-\nu) = 1$. The values of $\alpha(1-B)$ presented in (10) and (11) are to large. Thus the thermodynamic limit is not reached for $N = 1000$. For larger values of N, Q may still decrease. If it reaches asymptotically a nonzero value system-spanning and non-system-spanning slips would be equally important. But there is also the possibility that Q increases again and reaches one in the thermodynamic limit. In that case the system-spanning slips would dominate the averaged slip size. Nevertheless, as long as α is larger than one the locally driven FK model shows SOC in the sense that a power-law occurs which has a cutoff which diverges with the system size.

Figure 3 shows the pulling force (i.e., the force in the first spring just behind the pulled zeroth particle) as a function of time. The time is indirectly denoted by x_0 because of the infinitesimally slow pulling speed (i.e., $v \to 0$). Thus the slips are infinitely steep. In the attractor the averaged pulling force is just N times the mean local stress $\langle f\rangle$. The power spectrum of the pulling force shows a clear $1/f$-noise. The typical $1/f^2$ behavior of the jumps is found at frequencies above the averaged slipping frequency which is denoted by an arrow in Fig. 3.

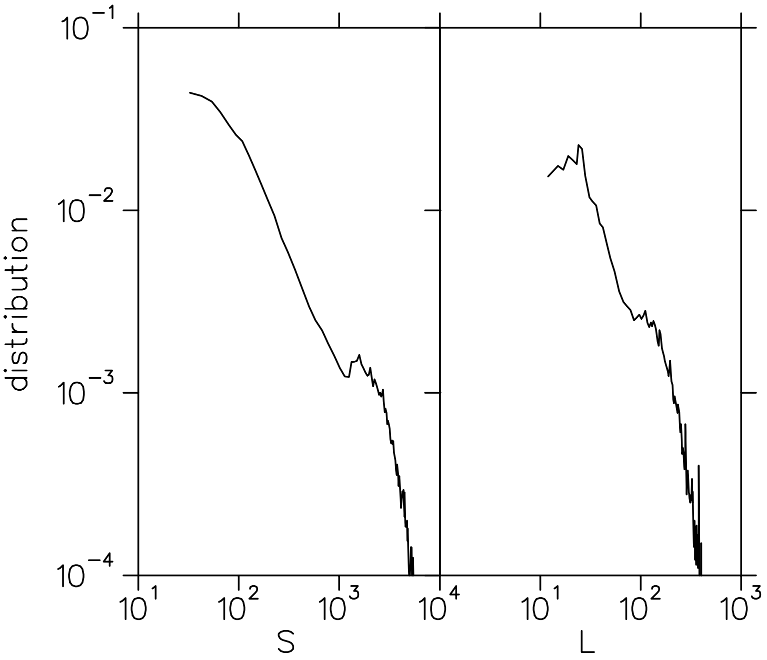

Figure 4. The FKT model: Distributions of the slip size S and the slip length L for $N = 500$, $\kappa = 0.01$, $c = 1.618$, $b = 1$, $g = 0.5$. The data is obtained from a simulation with $2 \cdot 10^4$ slips. The distribution for S is shifted upwards by one decade.

It should be remarked that it is not useful to measure the slip sizes in terms of the heights of the jumps in Fig. 3. The reason is that the height is roughly given by the the number of potential wells over which particle $j = 1$ travels during a slip. Thus the distribution is strongly modulated by the periodicity of the potential.

What is the reason for SOC in the locally driven FK model? First of all there is no intrinsic length scale except the "microscopic" one given by the periodicity of the external potential. The general belief [2] is that a system which organizes itself in a critical state is just able to transform a local disturbance into a global one. The locally driven FK model fits in this picture (for details see [13]).

4.4. THE FRENKEL-KONTOROVA-TOMLINSON (FKT) MODEL

The Frenkel-Kontorova-Tomlinson (FKT) model is a combination of the FK model and the Tomlinson model. The Tomlinson model or independent oscillator model [16] is a harmonic oscillator moved through a spatially periodic potential. It provides a realization of an idea of Tomlinson [17] which says that finite friction for infinitesimally small sliding velocities is conceivable by assuming that a molecule at the surface can be "plucked" by the other surface. The plucked molecule vibrates and dissipates its kinetic energy into the bulk. This dissipation is the reason for finite friction.

The FKT model is the counterpart of the BK model (see Fig. 1). The equations of motion read

$$\ddot{x}_j + g\dot{x}_j = x_{j+1} - 2x_j + x_{j-1} + \kappa(vt + cj - x_j) - V_E'(x_j), \quad j = 1, \ldots, N, \quad (17)$$

were c is the spacing of the support of the upper springs. In the BK model this spacing was irrelevant but in the FKT model it may be important because there is a competition of to different "microscopic" scales defined by the lattice constant of the upper surface given by c and the lattice constant of the lower surface given by the periodicity of V_E. Again I have chosen the piecewise parabolic potential (6) in order to do the limit $v \to 0$. Furthermore, I have chosen free ends, i.e., $x_0 = x_1 - c$ and $x_{N+1} = x_N + c$.

Here I present only preliminary results mainly to show that the FKT model, like the BK model, does not organize itself into a critical state. For that reason I have fixed the following parameters: $b = 1$, $c = 1.618$, and $g = 0.5$. Figure 4 shows the distributions of the slip size S and the slip length L for $\kappa = 0.01$ and $N = 500$. As for the BK model two different types of slips can be distinguished, localized ones and delocalized ones. The cross-over is at $L = \tilde{L} \approx 100$ and $S = \tilde{S} \approx 10^3$. These cross-over values are the same for $N = 1000$. The distributions of the localized slips seem to be power laws but only over a small range. The exponents are 1.4 and 1.9 for S and L, respectively.

For larger values of κ the cross-over is shifted to smaller values. I have done simulations for $\kappa = 10$ and 1. For these values of κ the cross-over has already disappeared. For $\kappa = 1$ only the exponential tail seen in Fig. 4 at very large slips has survived. The probability for slips larger than a few dozens of particles becomes very small.

Thus the FKT model shows irregular stick-slip motion but does *not* organizes itself into a critical state.

5. Summary and Concluding Remarks

I have reviewed four experiments which are motivated by the concept of SOC. Some experiments show slip-size distributions which are power laws (exponents ranging from 1.5 to 2) with a cutoff for large slip sizes. The nature of this cutoff has not been clarified except in the Ciliberto-Laroche experiment [8] where it was a clear finite-size cutoff. Only this experiment clearly shows SOC.

I have also reviewed numerical studies of four models. Only two of them, the train model and the FK model, show SOC. The exponents of the power laws in the slip-size distributions are in both models roughly the same (between 1.5 and 1.6). They are in agreement with the exponents found in experiments. The BK model and the FKT model have similar slip-size distributions but they do not show SOC.

The behavior of the Burridge-Knopoff-like models and the Frenkel-Kontorova-like models are very similar. The type of friction mechanism is not important. Whether a phenomenological friction law is given or the friction is modeled by a periodic potential plus an ordinary damping term does not seem to be essential. The external potential in the FKT model can even be replaced by a random potential without changing the overall behavior [18]. Only the effective kinetic friction should be much smaller than the static friction. Otherwise creeping instead of stick-slip motion is expected.

In all these models the dispersion relation of the phonons of the chain seems to be very important for the occurrence of SOC. In the train model and the FK model the dispersion relation is linear. It is structureless and defines no length scale. On the other hand the dispersion relation of the BK model and the FKT model has a gap. For wave numbers larger than a typical one it asymptotically approaches the linear one. This defines a intrinsic *macroscopic* length scale which destroys SOC.

The BK model and the FKT model seem to be more realistic models of sliding surfaces than the train model or the FK model where the springs are strongly stretched and may eventually break. Nevertheless the exponents of the power laws found in the experiments are similar to the exponents found for the train model and the FK model. On the other hand the BK model and the FKT model are also unrealistic. If the chain mimics a lubricant the upper springs are not a realistic description of the interaction. If the blocks or particles mimic asperities it is unrealistic to assume very weak asperities but a very stiff support (i.e., a rigid body). Thus in any case the gap in the phonon dispersion is artificial. Therefore the intrinsic length scale is not real. A more realistic model which avoids such artificial length scales is worth to be studied in order to find out whether SOC in dry friction can be a generic behavior.

Like phase transitions SOC is a behavior which emerges in the thermodynamic limit. But a majority of tribological systems are not working in this limit because the number of micro contacts is small (of the order $10-100$ [1]). They are randomly forming and breaking which is most often the source of irregularities in the stick-slip motion. In order to get SOC it is absolutely necessary to have a large number of meta-stable states. For that reason the two conditions mentioned at the end of Sec. 2 are crucial: Elastic sliding bodies and a large number of micro contacts.

Is SOC possible in dry friction? Yes, it is possible. But for most friction experiments the answer is no. Thus a practical tribologist does not have to learn this new concept of self-organized criticality in order to understand the majority of irregular stick-slip motion.

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